PARALLELIZED MULTILEVEL CHARACTERISTIC BASIS FUNCTION METHOD (MLCBFM) COMBINED WITH ADAPTIVE CROSS APPROXIMATION (ACA) FOR THE ANALYSIS OF THE SCATTERING FROM ELECTRICALLY LARGE ROUGH SURFACES

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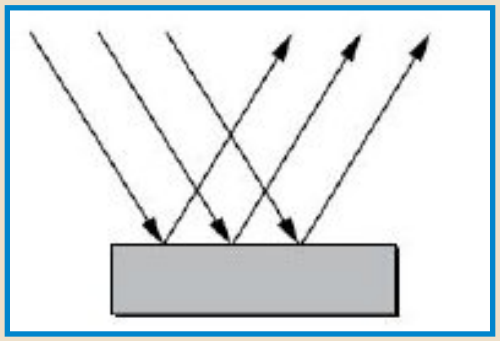
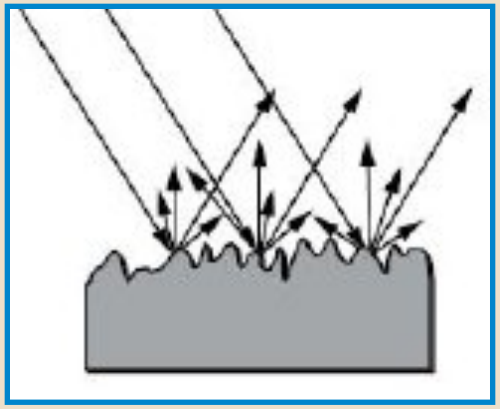


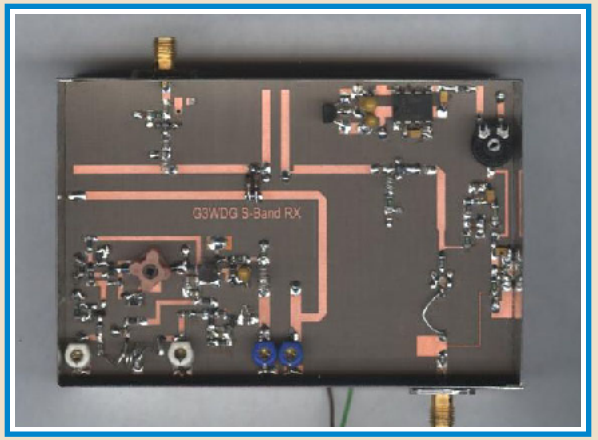


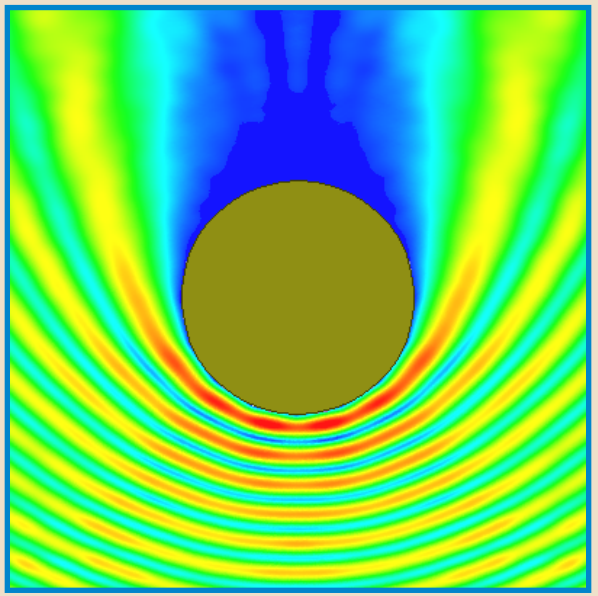
MOTIVATION (1)

❖The **Method of Moments (MoM)** is a widely used algorithm for the electromagnetic analysis of tridimensional scattering as well as circuit problems

❖ The MoM associated matrix grows rapidly as the problem electrical dimensions become large or if a finite mesh is used to model complex structures in order to guarantee a reasonable solution accuracy

❖ The memory allocation and the CPU time increase as $O(N^2)$ and $O(N^3)$ if N is the total number of unknowns. This makes difficult the solution of electrically large problems by using the MoM



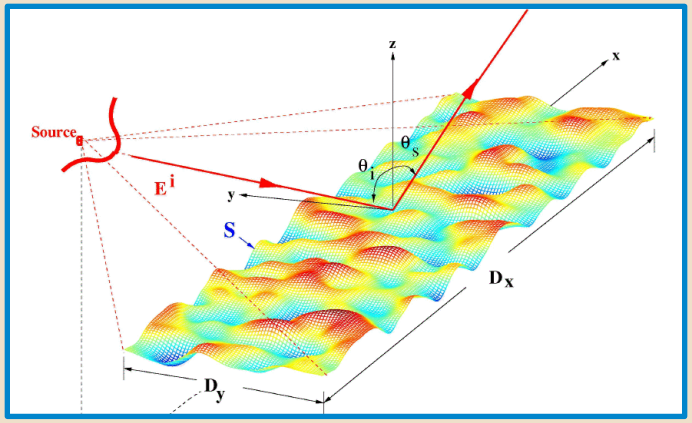


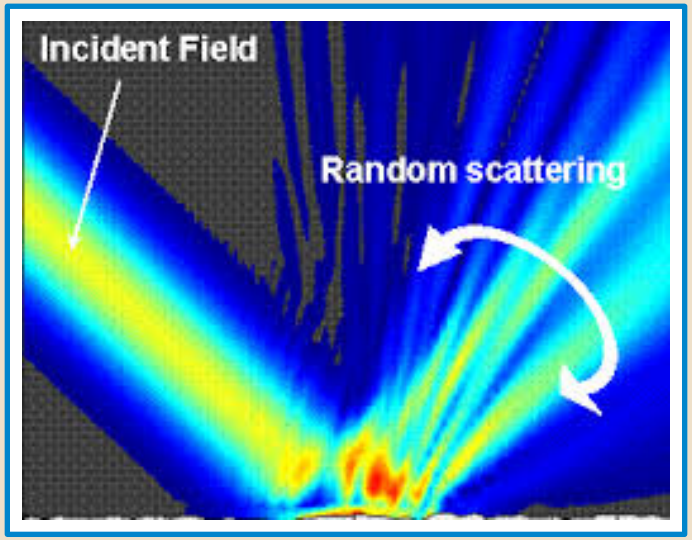
MOTIVATION (2)

❖The main scope of this presentation is the scattering analysis of the perfectly conducting **Rough Surfaces** via the Method of Moments

❖ The electromagnetic modeling of rough surfaces often requires several hundreds thousands of low level basis functions (RWGs or Rooftops)

❖ Conventional iterative solvers, such as the BICGSTAB(l) [R_1] are characterized by converge issues for plane waves shining the surface from low-grazing angles or are very timing consuming for multiple RHS problems as the mono-static RCS type of analysis



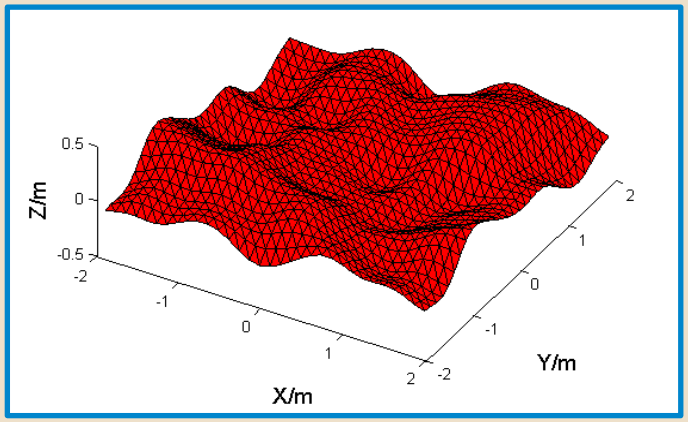


[R_1] E. Topsakal, R. Kindt, K. Sertela and J. Volakis "Evaluation of the BICGSTAB(l) algorithm for the finite-element/boundary-integral method," *Antennas and Propagation Magazine, IEEE* , vol.43, no.6, pp. 124-131, Dec 2001

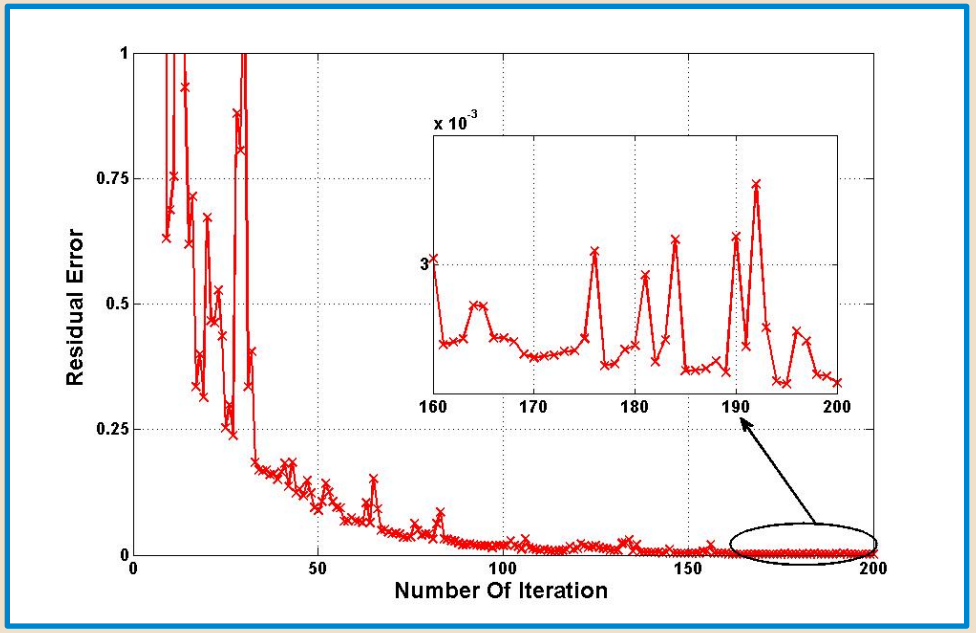
MOTIVATION (3)

❖ We have analyzed one realization of a rough surface by using a conventional iterative solver:

Surface Realization:



Converge History:

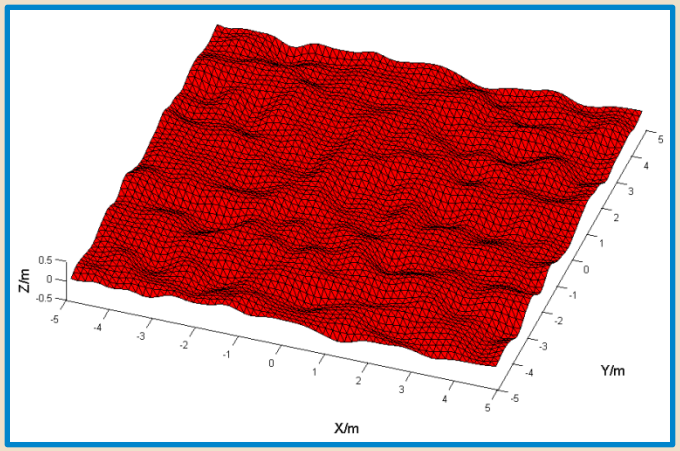


❖ Iterative solver: BICGSTAB
❖ Surface size: 16.0λ×16.0λ
❖ Frequency: 300.0 MHz
❖ Number of RWGs: 53133
❖ Number of excitations: 160
❖ Solution type of 1 incident angle: 640s (Incident angle (θ_i, ϕ_i): (70, 0)deg)
❖ Mono-static RCS solution time: 95421s (26.5 h)

ROUGH SURFACE STATISTICAL PROPERTIES

❖ The analyzed rough surfaces are characterized by a Gaussian correlation function *C(x,y)* and by a Gaussian power spectral density distribution $W(k_x,k_y)$ [R_2]

Surface Realization:



$$\langle f(x_1,y_1), f(x_2,y_2)\rangle = hC(|x_1-x_2|,|y_1-y_2|)$$

$$C(x,y) = \exp\left(-\frac{(x^2+y^2)}{l^2}\right)$$

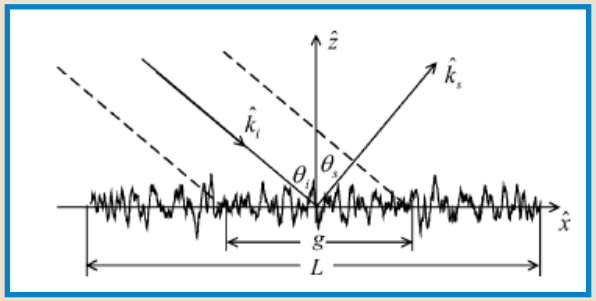
$$W(k_x,k_y) = \left(\frac{lh}{2\sqrt{\pi}}\right)^2 \exp\left(-\frac{l^2(k_x{}^2+k_y{}^2)}{4}\right)$$

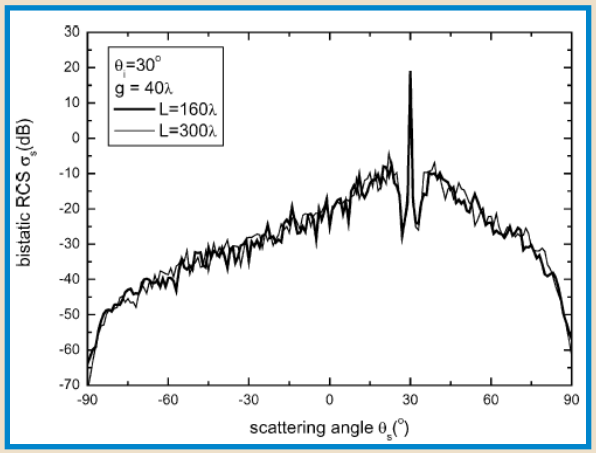
- ***C(x,y):*** correlation function
- ***$W(k_x,k_y)$:*** spectral density distribution
- ***l:*** correlation length
- ***h:*** profile RMS
- ***f(x,y):*** surface profile

[R_2] X. Gu, L. Tsang and H. Braunisch, "Modeling Effects of Random Rough Interface on Power Absorption Between Dielectric and Conductive Medium in 3-D Problem," *Microwave Theory and Techniques, IEEE Transactions on*, vol. 55, no. 3, pp. 511-517, March 2007

TAPERED PLANE WAVES

❖ In order to avoid the edge effects from the "finite" analyzed surface, the tapered plane wave concepts reported in [R_3] has been implemented





$$\underline{E}_i = \exp\left[\left(-j\underline{k}\cdot\underline{r}\right)(1+w)\right]\exp\left[-\left(t_x + t_y\right)\right]$$

$$t_x = \frac{\underline{r}\cdot\hat{\vartheta}}{\left(g\cos\vartheta_i\right)^2} \qquad t_y = \frac{\underline{r}\cdot\hat{\phi}}{g^2}$$

$$w = \frac{1}{k^2}\left[\frac{2t_x - 1}{\left(g\cos\vartheta_i\right)^2} + \frac{2t_y - 1}{g^2}\right]$$

❖ The tapering parameter *g* must be chosen according to the rough surface length *L*

❖ Small *g* values will produce inaccurate results relative to the specular reflection direction

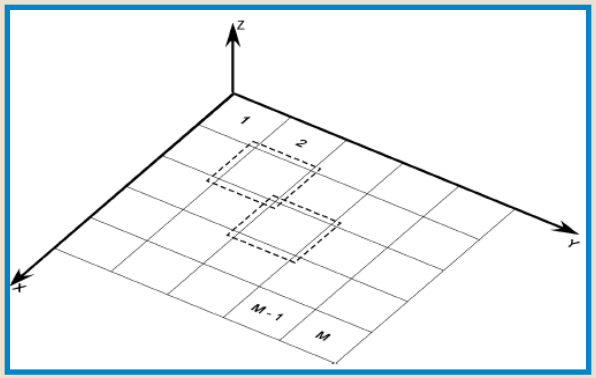
❖ For all the results shown in this presentation we have chosen *g=L/4*

[R_3] H. Ye and Y.-Q. Jin, "Parameterization of the tapered incident wave for numerical simulation of electromagnetic scattering from rough surface," *Antennas and Propagation, IEEE Transactions on*, vol. 53, no. 3, pp. 1234-1237, March 2005

CHARACTERISTIC BASIS FUNCTION METHOD

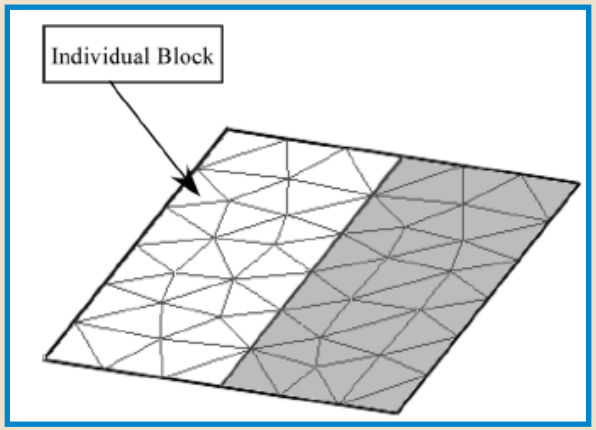
Step - I: The first step in **CBFM** is to divide the problem in smaller regions with respect to the original geometry, called “blocks” [R_4]

❖ The induced currents in each blocks are represented by using a type of high-level basis function defined **Primary Basis Functions**



Step - II: The **Primary Basis Functions** are evaluated by analyzing each block in the absence of the others:

$$\underline{\underline{Z}}_{ii} \cdot \underline{P}_{i,n} = \underline{V}_{i,n} \qquad \begin{matrix} i = 1,2,\cdots,M \\ n = 1,2,\cdots,N \end{matrix}$$



[R_4] E. Lucente, A. Monorchio and R. Mittra, "Parameterization of the tapered incident wave for numerical simulation of electromagnetic scattering from rough surface," *Antennas and Propagation, IEEE Transactions on*, vol. 56, no. 4, pp. 999-1006, April 2008

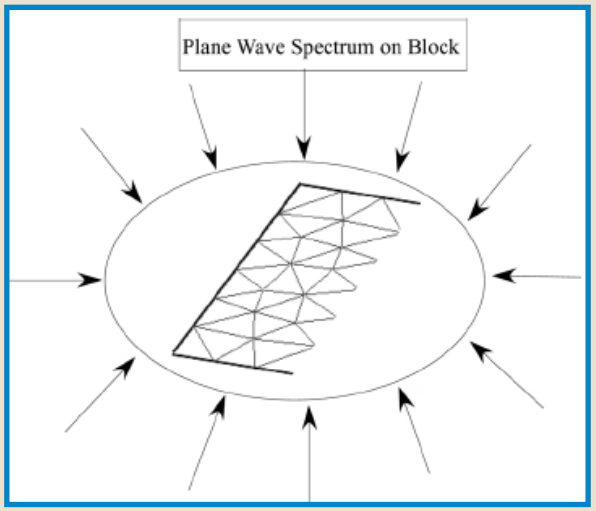
CHARACTERISTIC BASIS FUNCTION METHOD

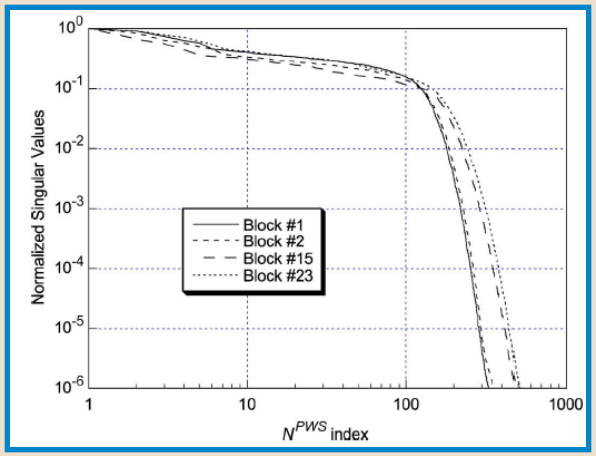
❖ Each block is excited by using a **Spectrum Of Plane Waves (SPWs)** intentionally overestimating the **Degrees Of Freedoms (DOFs)** of the problem

Step - III: After generating the CBFs, a thresholding procedure based on the a **Singular Value Decomposition (SVD)** approach, can be employed in order to reduce the number of CBFs and the reduced matrix size

Step - IV: The final current distribution ***J*** induced on the scatterer can be expressed as a linear combination of the **CBFs** provided by the **SVD** procedure ($\underline{J}_{n,m}$):

$$\underline{J} = \sum_{n=1}^{N_1} \alpha_{1,n} \underline{J}_{1,n} + \sum_{n=1}^{N_2} \alpha_{2,n} \underline{J}_{2,n} + \cdots + \sum_{n=1}^{N_M} \alpha_{M,n} \underline{J}_{M,n}$$





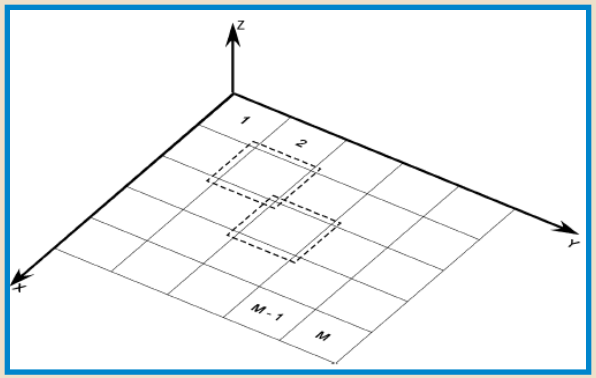
CHARACTERISTIC BASIS FUNCTION METHOD

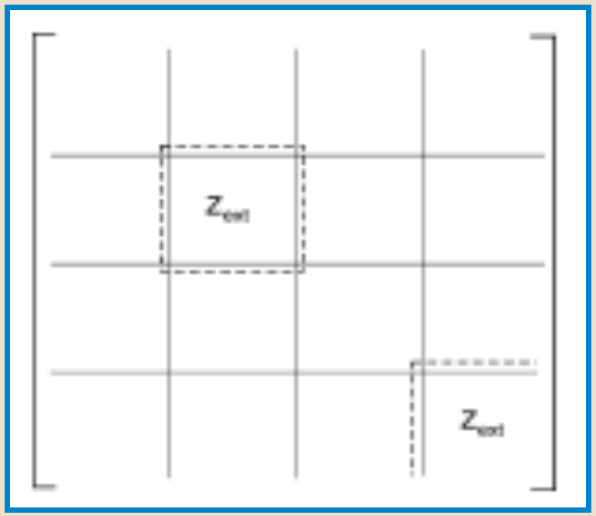
Step – V: The final step to be performed is the generation of the **Reduced matrix Z^r** that can be accomplished by applying the Galerkin testing procedure employing the CBFs as testing functions

$$\underline{\underline{Z}}^r \cdot \underline{\alpha} = \underline{V}^r \quad \underline{\underline{Z}}^r = \underline{J}^T \cdot \underline{\underline{Z}} \cdot \underline{J} \quad \underline{V}^r = \underline{J}^T \cdot \underline{V}$$

Z: MoM impedance matrix
J: matrix comprising all the CBFs after the SVD procedure
V: RHS vector related to the real problem excitations

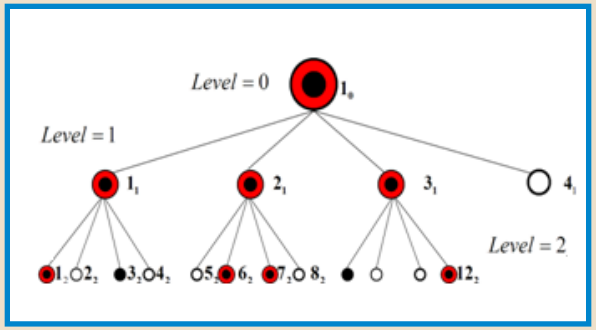
❖ The CBFM leads to a reduced matrix, which is much smaller than the original one, and this obviates the need for an iterative solution of problems requiring a large number of unknowns





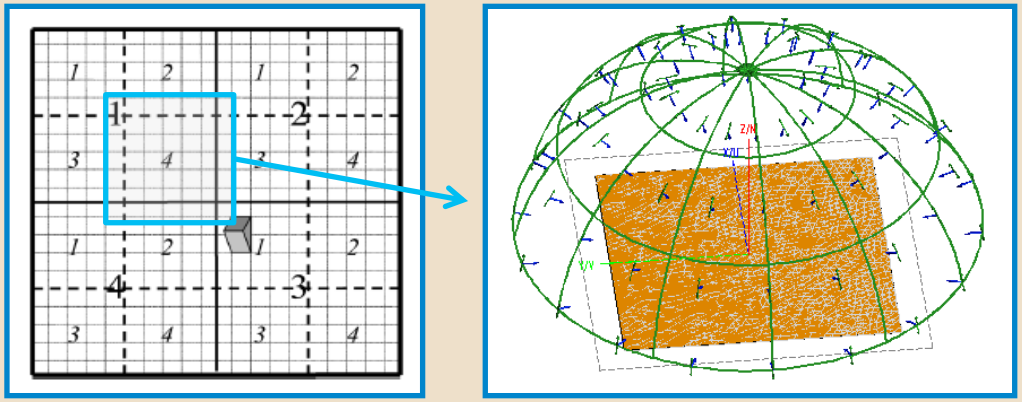
MULTILEVEL CBFM

❖ In this paper a **Multilevel CBFM (MCBFM)** has been implemented in order to efficiently solve MoM problems characterized by more than 100.000 low-level basis function [R_5]



❖ The compression rate between low-level and high-level basis functions increases as the size of the block and, hence, the number of basis functions inside it, is increased

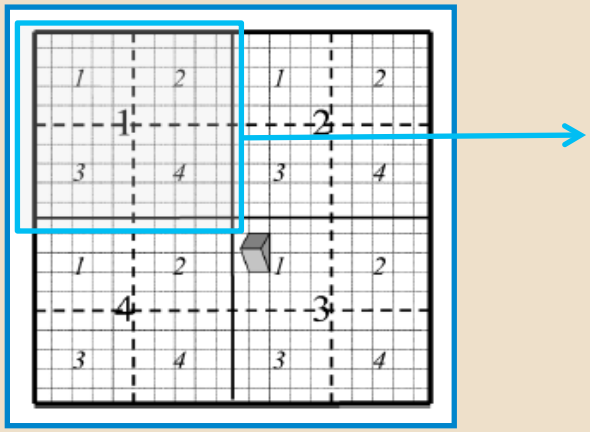
❖ The CBFs at the first level are constructed by using a spectrum of plane waves with a wide range of incident angles



[R_5] J. Laviada, F. Las-Heras, M. R. Pino and R. Mittra, "Solution of Electrically Large Problems With Multilevel Characteristic Basis Functions," *Antennas and Propagation, IEEE Transactions on*, vol. 57, no. 10, pp. 3189-3198, October 2009

MULTILEVEL CBFM

❖ To determine the higher level CBFs, the lower level blocks are hierarchically grouped together



$$\underline{\underline{Z}}_{i,i}^{(q)} = \begin{bmatrix} \underline{\underline{Z}}_{1,1}^{(q-1)} & \underline{\underline{Z}}_{1,2}^{(q-1)} & \cdots & \underline{\underline{Z}}_{1,K_i^{(q-1)}}^{(q-1)} \\ \underline{\underline{Z}}_{2,1}^{(q-1)} & \underline{\underline{Z}}_{2,2}^{(q-1)} & \cdots & \underline{\underline{Z}}_{2,K_i^{(q-1)}}^{(q-1)} \\ \vdots & \vdots & \ddots & \vdots \\ \underline{\underline{Z}}_{K_i^{(q-1)},1}^{(q-1)} & \underline{\underline{Z}}_{K_i^{(q-1)},2}^{(q-1)} & \cdots & \underline{\underline{Z}}_{K_i^{(q-1)},K_i^{(q-1)}}^{(q-1)} \end{bmatrix}$$

$$\underline{\underline{Z}}_{i,j}^{(q)} = \left\langle \left(\underline{\underline{J}}_{i,i}^{(q-1)} \right)^t \underline{\underline{Z}}_{i,j}^{(q-1)} \underline{\underline{J}}_{i,i}^{(q-1)} \right\rangle$$

❖ Once the higher level expansion coefficients have been determined, the final current distribution can be uncompressed in an recursive manner

$$\underline{J}(\underline{r}) = \sum_{n=0}^{N-1} \alpha_n^{(Q-1)} \underline{J}_n^{(Q-1)}(\underline{r}) \quad \underline{J}_{n,m}^{(q)}(\underline{r}) = \sum_{p=0}^{N_n^{(q-1)}} \sum_{l=0}^{K_{n,p}^{(q-1)}} \alpha_l^{(q-1)} \underline{J}_{p,l}^{(q-1)}(\underline{r})$$

THE ADAPTIVE CROSS SECTION ALGORITHM

❖ The **Adaptive Cross Section Algorithm (ACA)** has been implemented in order to efficiently calculate the reduced matrix at each level of the proposed algorithm [R_6]

❖ For well-separated groups of RWGs, the electric field at the observation group produced by any source RWG can be expressed as a linear combination of the fields produced by only a few of these source RWGs

❖The ACA algorithm is purely algebraic in nature and can be used irrespectively of the kernel of the integral equation, basis functions or type of integral equation formulation

$$\underline{\underline{Z}}^{r}_{i,j} = \left\langle \underline{\underline{J}}^{t}_{ii} \underline{\underline{Z}}^{RWG}_{i,j} \underline{\underline{J}}_{jj} \right\rangle \approx \left\langle \underline{\underline{J}}^{t}_{ii} \underline{\underline{\tilde{Z}}}^{RWG}_{i,j} \underline{\underline{J}}_{jj} \right\rangle$$

$$\underline{\underline{\tilde{Z}}}^{RWG}_{i,j} = \underline{\underline{U}}^{K_p \times r}_{i} \cdot \underline{\underline{V}}^{r \times K_p}_{i}$$

$$\left\| \underline{\underline{R}} \right\|_F = \left\| \underline{\underline{Z}}^{RWG}_{i,j} - \underline{\underline{\tilde{Z}}}^{RWG}_{i,j} \right\|_F$$

$$\left\| \underline{\underline{R}} \right\|_F \le \kappa \left\| \underline{\underline{Z}}^{RWG}_{i,j} \right\|_F$$

$$\left\| \underline{\underline{R}} \right\|_F = \sqrt{\sum_{n=0}^{N-1} \sum_{m=0}^{M-1} \left| R_{i,j} \right|^2}$$

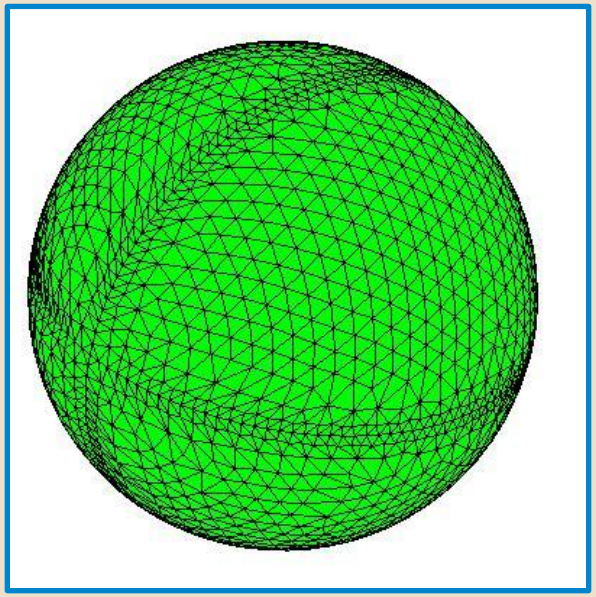
[R_6] R. Maaskant, R. Mittra and A. Tijhuis, “Fast Analysis of Large Antenna Arrays Using the Characteristic Basis Function Method and the Adaptive Cross Approximation Algorithm" *Antennas and Propagation, IEEE Transactions on*, vol. 56, no. 11, pp. 3440-3451, November 2008

CODE VALIDATION

❖ The developed code has been validated by analyzing the canonical example of a Perfect Electric Conductor (PEC) sphere

❖ The results provided by the MCBFM have been compared with the conventional CBFM ones and with the Mie solution

❖ The code accuracy have been tested by using the reported Relative Error definition

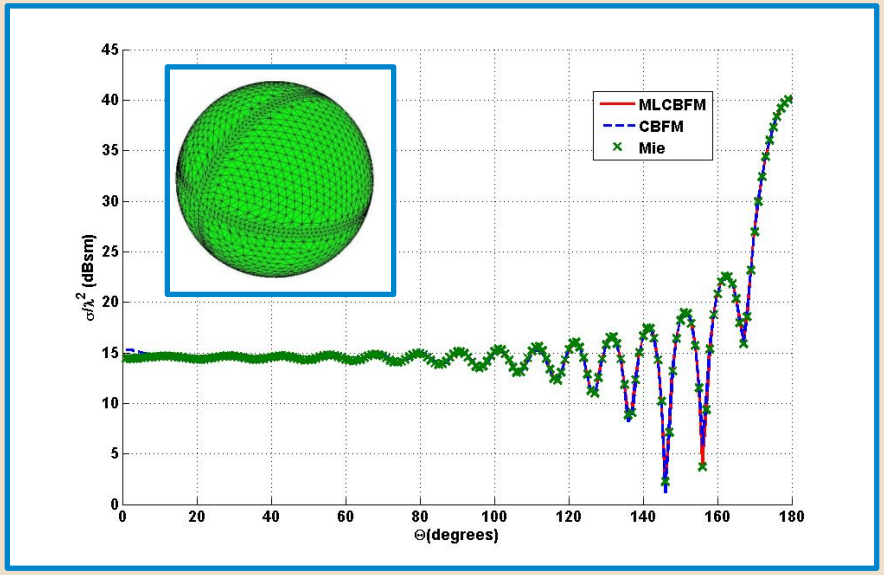


❖ Sphere radius: 3.0m
❖ Frequency: 300MHz
❖ Number of RWGs: 52,023
❖ SVD threshold: $1e^{-3}$
❖ ACA threshold: $1e^{-3}$
❖ Number of levels in the MCBFM: 2

$$Rel.\ Error = 100 * \sum_{n} \frac{\left| RCS_{MCBFM,n} - RCS_{Mie,n} \right|}{\left| RCS_{Mie,n} \right|}$$

CODE VALIDATION

Bistatic RCS:



Performance summary:

	CBFM	MLCBFM
Number of blocks	64	16
Dimensions of the reduced matrix	8598	2932
Solution time(s):	551	171

$$Relative\ Error(\%) = 0.3327$$

❖ Very good agreement has been achieved between the proposed approaches

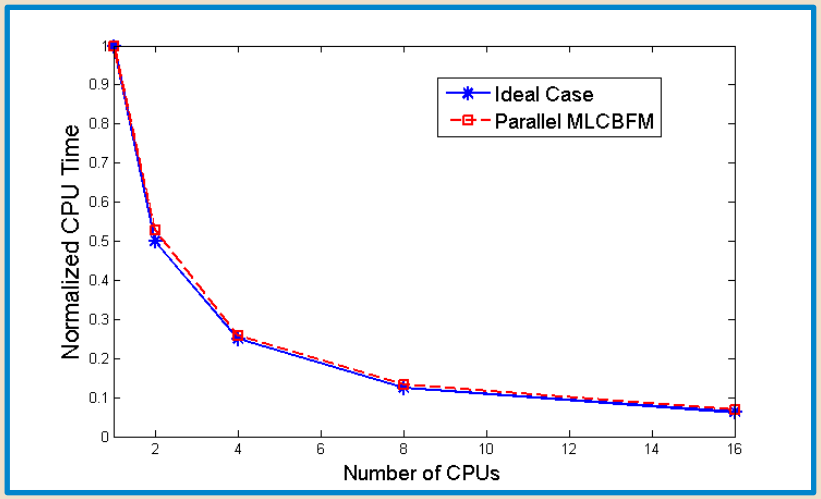
❖ The MLCBFM is characterized by a smaller reduced matrix which can be solved more efficiently. The reported time saving property is more evident when analyzing problems characterized by a larger number of low-level basis functions distributed in a small number of blocks

CODE VALIDATION

❖ We have also tested the numerical efficiency of the parallel algorithm by progressively increasing the number of CPUs employed to solve the sphere problem

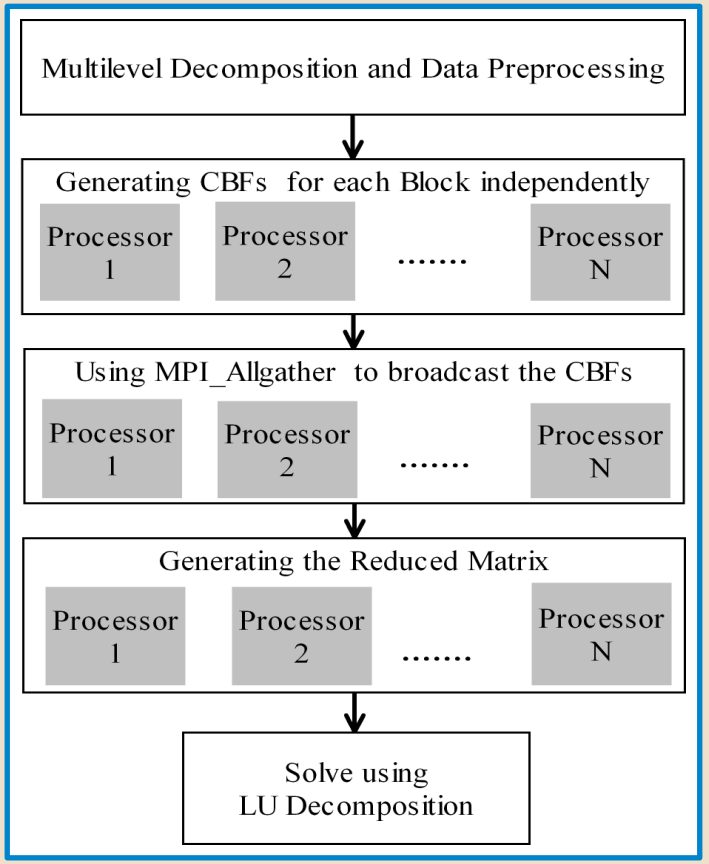
❖ The obtained efficiency has been compared with respect to the ideal case where the solution time scales as *1/n*

CPU Time:



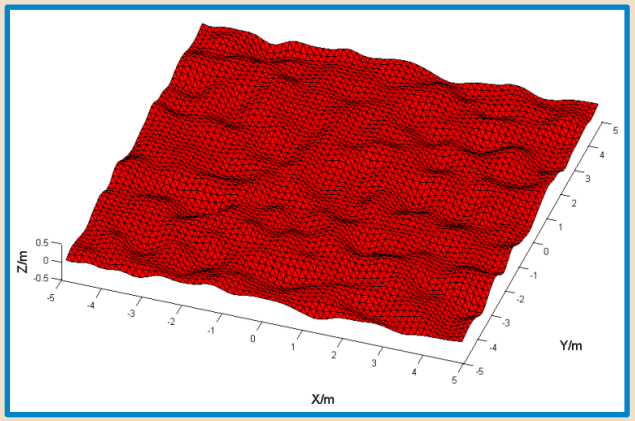
❖ The parallelized CBFM algorithm scales quite well

Parallel Implementation:



ROUGH SURFACES WITH MLCBFM (1)

Surface Realization:



Monte Carlo approach:

$$\eta_{RCS} = \frac{1}{N}\sum_{n=0}^{N-1} RCS_n$$

- We have analyzed a rough surface profile, whose characteristics have been reported below, by using the CBFM and the MLCBM

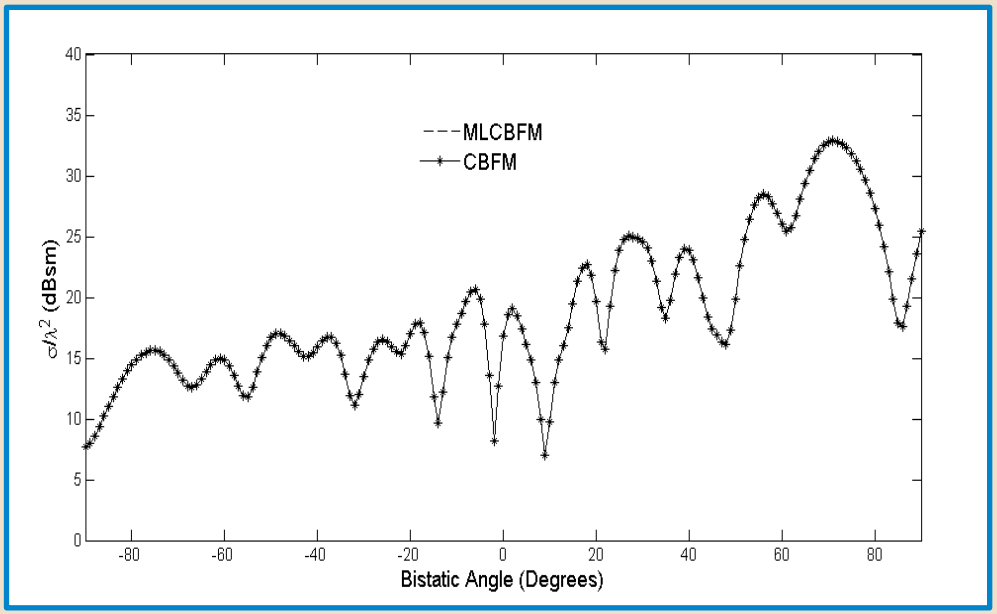
- Frequency: 300MHz
- Rough surface dimensions (m): 16x16
- Number of RWGs: 53133
- Power spectral density type: Gaussian
- Profile RMS height *h*: 0.2m
- Correlation length *l*: 1.6m
- Incident angle (θ_i, ϕ_i): (70, 0)°

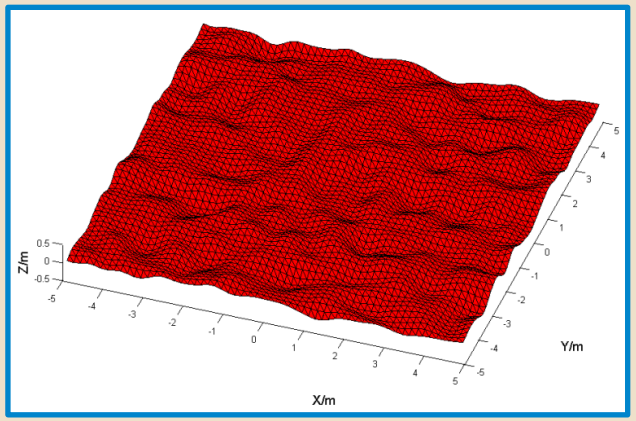
Gaussian Power Spectral Density:

$$W(k_x, k_y) = \left(\frac{lh}{2\sqrt{\pi}}\right)^2 \exp\left(-\frac{l^2\left(k_x^{\,2} + k_y^{\,2}\right)}{4}\right)$$

ROUGH SURFACES WITH MLCBFM (1)

Bistatic RCS:





Performance summary:

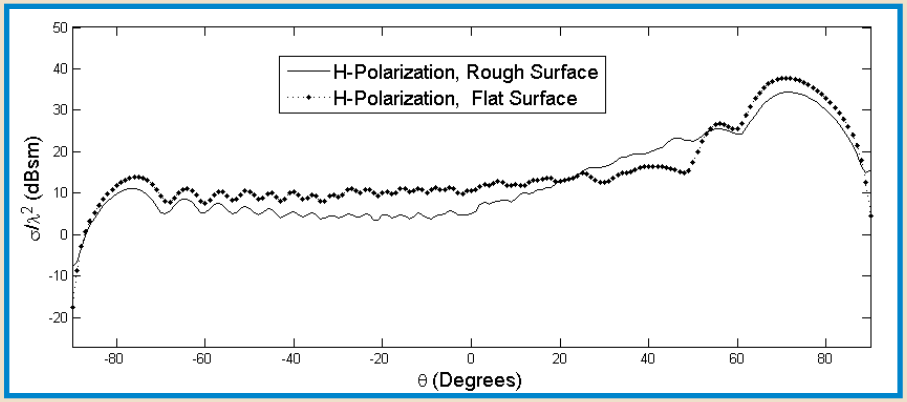
	CBFM	MLCBFM
Number of blocks	16	4
Dimensions of the reduced matrix	8832	3445
Solution time(s):	578	227

❖ Very good agreement has been achieved between CBFM and MLCBFM while the iterative solvers is characterized by……

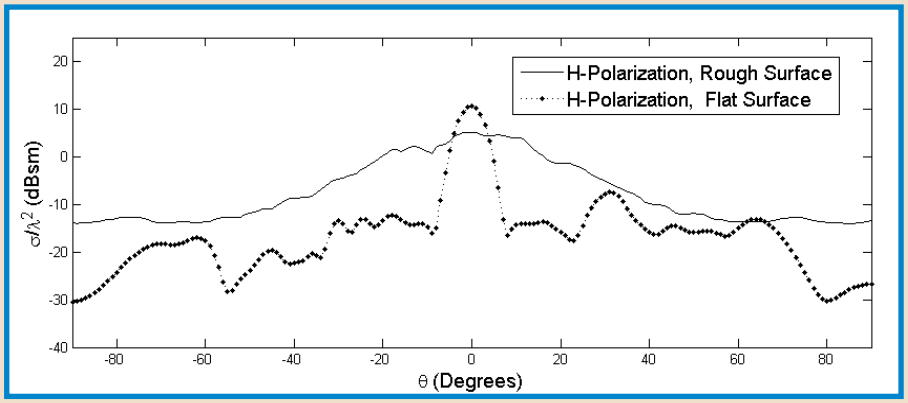
ROUGH SURFACES WITH MLCBFM (1)

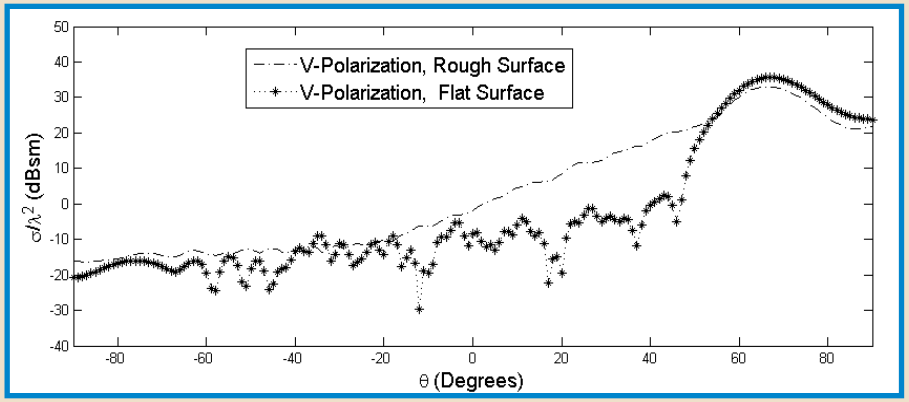
❖ We have calculated the RCS statistical distribution of the analyzed rough surface comparing the results with ones associated with its flat counterpart

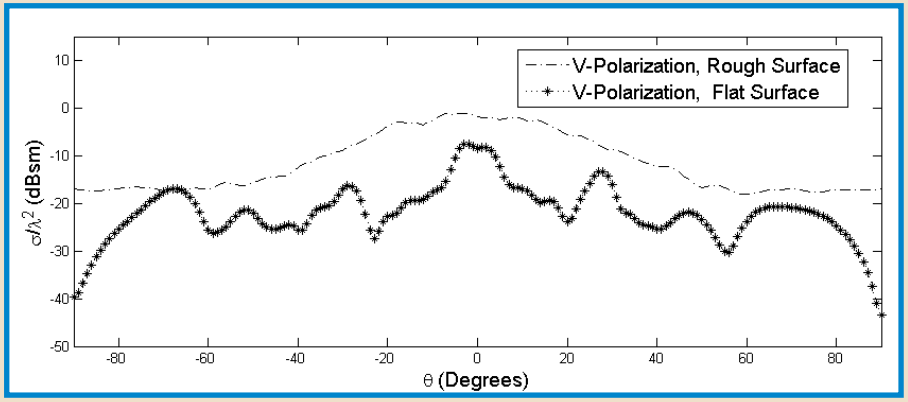
y-z plane:



x-z plane:

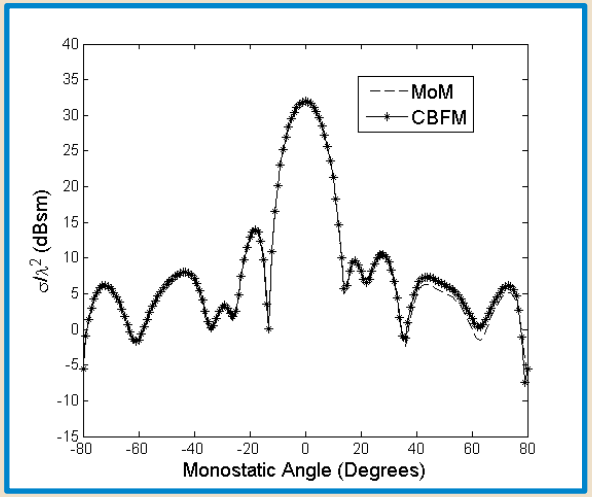




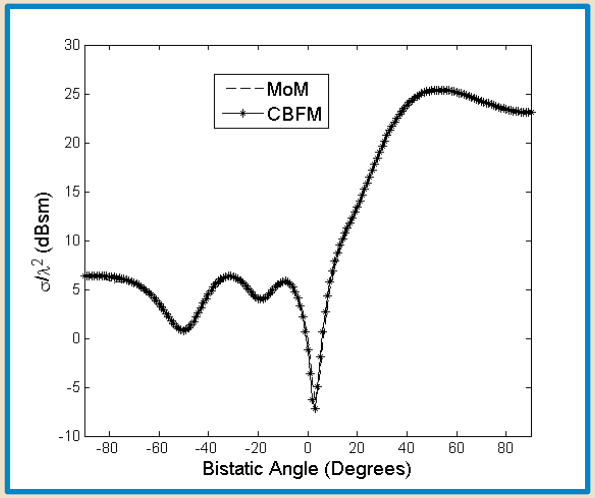


ROUGH SURFACES WITH MLCBFM (2)

Monostatic RCS



Bistatic RCS:

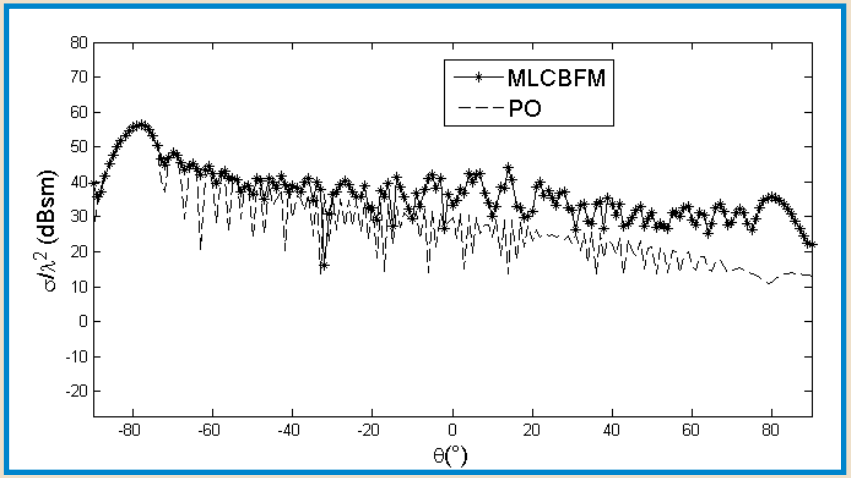


- The CBF generation is time consuming and memory demanding since they need to be generated anew for each frequency [R_7]
- Frequency sweep analysis can be made highly efficient by generating CBFs only for highest frequency of the band and used at lower frequencies too over 2:1 frequency band
- Scattering from a 8mx8m surface
- Frequency: 300MHz
- Frequency range (GHz): 0.3-0.6
- CBFs are generated at 0.6 GHz
- RCS is obtained at 0.3 GHz
- Incident angle (θ_i, ϕ_i): (70, 0)°

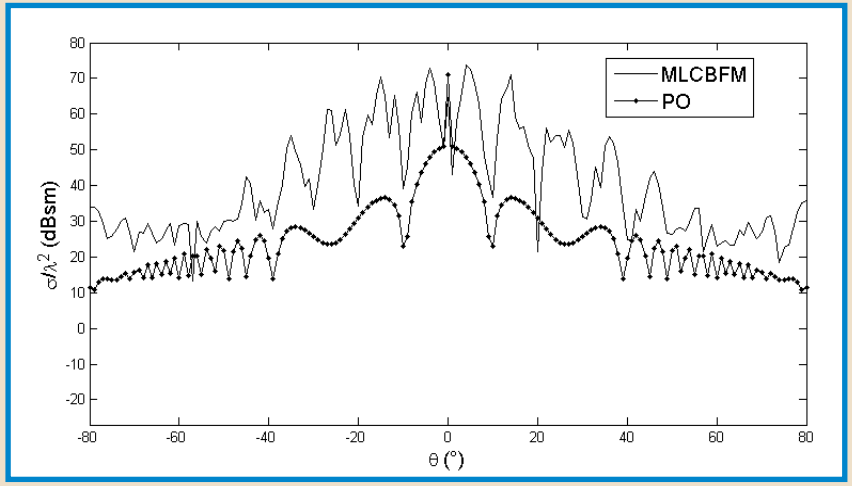
[R_7] M. De-Gregorio, G. Tiberi, A. Monorchio and R. Mittra, "Solution of wide band scattering problems using the characteristic basis function method" *Microwaves, Antennas & Propagation, IET* , vol. 6, no. 1, pp. 60-66, Jannuary 2012

ROUGH SURFACES WITH MLCBFM (3)

Bistatic RCS:



Monostatic RCS:



❖ We have analyzed a rough surface profile, whose characteristics have been reported below, by using the CBFM, the MLCBM and the conventional PO

- ❖ Frequency: 300MHz
- ❖ Rough surface dimensions (m): 32x32
- ❖ Number of RWGs: 194565
- ❖ Power spectral density type: Gaussian
- ❖ Profile RMS height *h*: 0.1m
- ❖ Correlation length *l*: 0.5m
- ❖ Incident angle (θ_i, ϕ_i): (80, 0)°

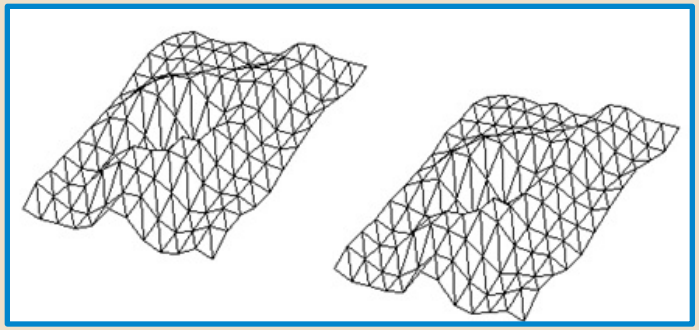
	CBFM	MLCBFM
Number of blocks	64	8
Dimensions of the reduced matrix	18528	8284
Solution time(s):	1763	556

ROUGH SURFACES WITH MLCBFM (4)

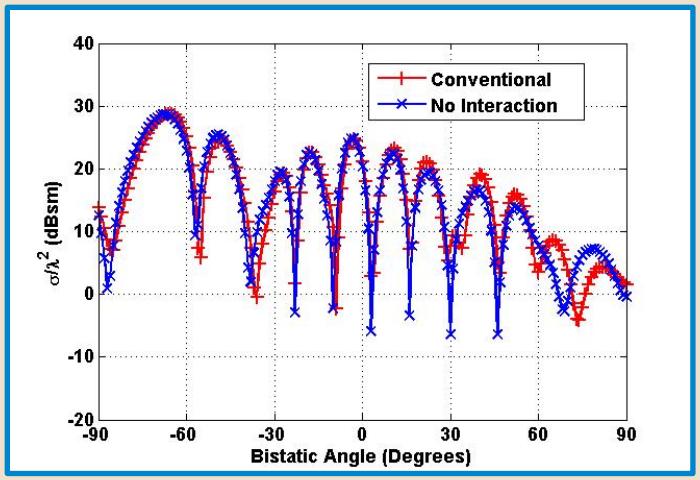
❖ We have tested the possibility of neglecting EM interaction between blocks separated by an electrical distance grater a than a certain threshold

❖ Rough surface dimensions (λ): 4x4
❖ Separation distance (λ): 2
❖ Number of RWGs: xxxxx
❖ Power spectral density type: Gaussian
❖ Profile RMS height *h*: xxxxx
❖ Correlation length *l*: xxxxx
❖ Incident angle (θ_i, ϕ_i): (0, 0)°

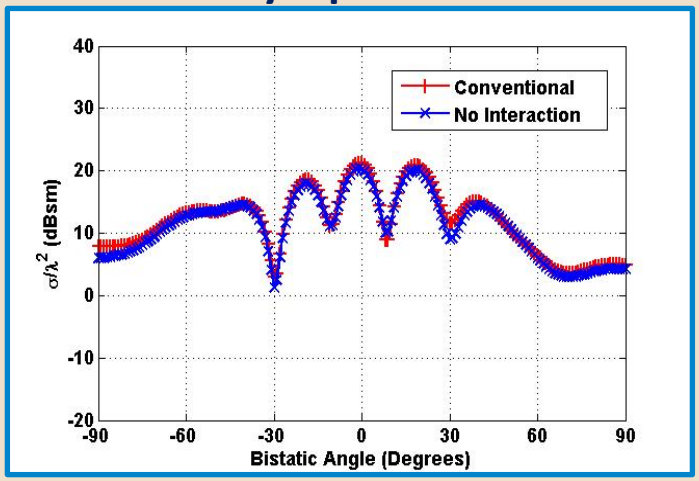
Surface Realization:



x-z plane:

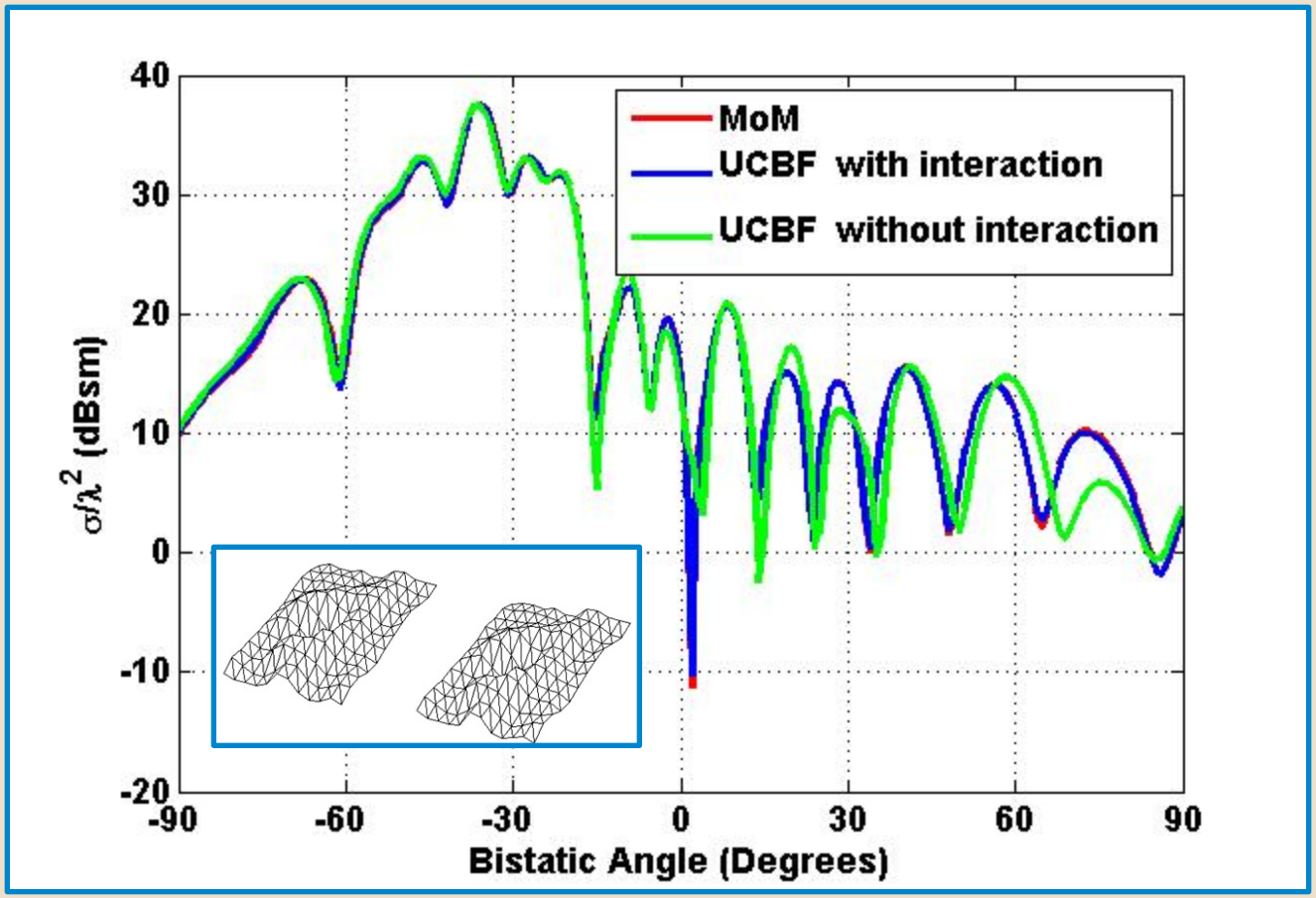


y-z plane:



ROUGH SURFACES WITH MLCBFM (4)

- We have calculated the RCS statistical distribution of the analyzed rough surface
- The CBFs have been generated at 0.6GHz
- The RCS distribution has been calculated at 0.3GHz

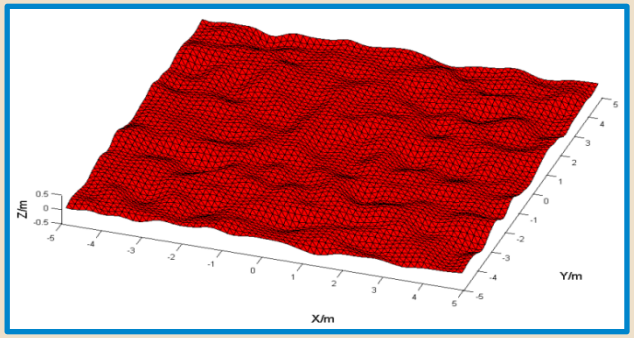


ROUGH SURFACES WITH MLCBFM (4)

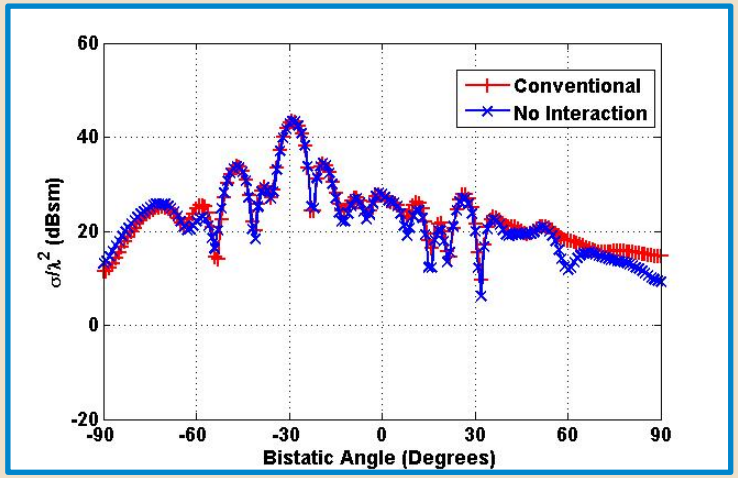
❖ We have tested the possibility of neglecting EM interaction between blocks separated by an electrical distance grater a than a certain threshold

- ❖ Rough surface dimensions (λ): 16x16
- ❖ Number of RWGs: xxxxx
- ❖ Power spectral density type: Gaussian
- ❖ Profile RMS height *h*: xxxxx
- ❖ Correlation length *l*: xxxxx
- ❖ Incident angle (θ_i, ϕ_i): (0, 0)°
- ❖ Separation distance threshold (λ): 4

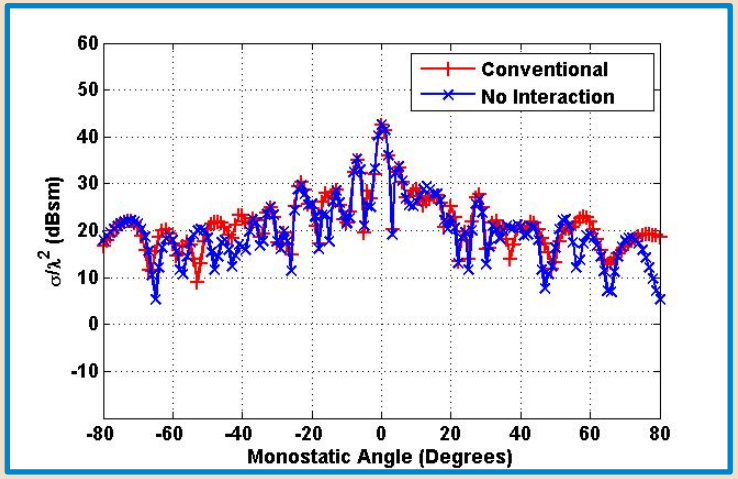
Surface Realization:



Bistatic:



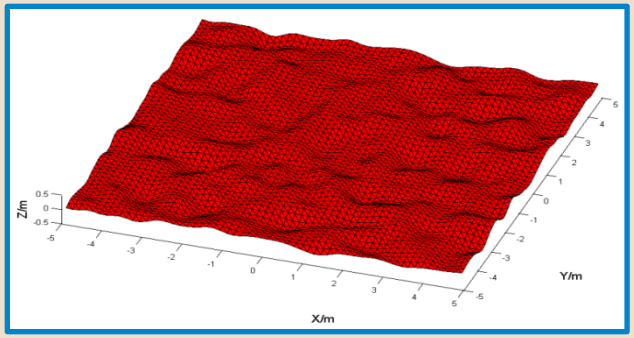
Monostatic:



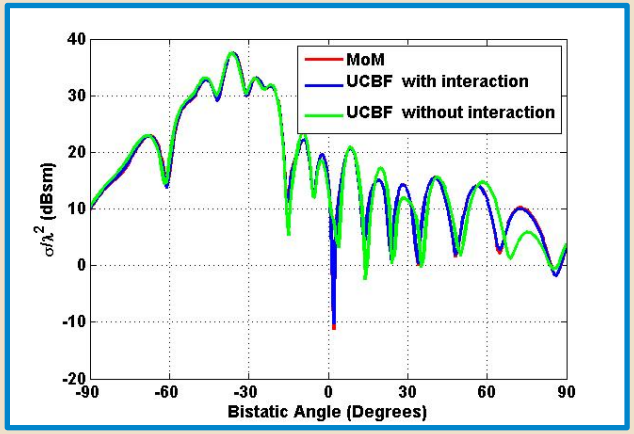
CONCLUSIONS AND FUTURE DEVELOPMENTS

❖ The Multilevel Characteristic Basis Function Method has been applied to analysis of the electromagnetic scattering from randomly generated rough surfaces

❖ The MLCBFM is an iteration-free, high parallelizable approach which can efficiently solve scattering problems characterized by multiple Right Hand Sides and grazing incident angles



❖ In order to reduce the computational burden, the CBFs have been generated only at the highest frequency of interest and the electromagnetic interaction between blocks separated by an electrical distance above a certain threshold has been neglected



❖ Very good agreement has been achieved between the proposed MLCBFM and the conventional MoM, while the Physical Optic approach does not provide accurate results

❖ As a future development, we are currently investigating an efficient way to calculate the CBF interaction in the wave-number domain